# Minimum Spanning Trees

## Introduction

Minimum Spanning Trees (MSTs) are important data structures in Computer Science. Determining MSTs of a graph is an important application of greedy algorithms.

## Learning Objectives

By the completion of this topic, the student should:

* Understand the use of greedy methods to find MSTs. In particular:
  + Understand the idea of growing an MST by selecting “safe edges”, including why the loop invariant for Generic-MST(…) is important, and why it is satisfied.
  + Be able to grow an MST using Kruskal’s algorithm.
  + Be able to grow an MST using Prim’s algorithm.
* Be able to use greedy algorithms to find approximations for the optimal TSP tour of a graph.

## Books, Videos, Web Sites

### Books:

Cormen, Leiserson, Rivest, and Stein, *Introduction to Algorithms, 3rd ed.*, Chapte 23.

You should understand Kruskal’s and Prim’s algorithms and be able to use them. Do not worry about implementing Kruskal’s algorithm, efficient implementations use a data structure that we don’t cover.

We will also talk about a topic from Chapter 35, the MST approximation to the TSP problem. It’s sufficient if you just read the lecture notes at this time. We will cover approximations in detail the last week of the semester.

### Web Sites:

I like the geeks-for-geeks.org pages on [Greedy Algorithms](http://www.geeksforgeeks.org/fundamentals-of-algorithms/#GreedyAlgorithms). It includes pages on Kruskal’s and Prim’s algorithms for computing MSTs, and a page on [using MST algorithms to come up with a good (but not often optimal) solution to the TSP.](https://www.geeksforgeeks.org/travelling-salesman-problem-set-2-approximate-using-mst/).

[Hackerearth.com has a good page on MSTs.](https://www.hackerearth.com/practice/algorithms/graphs/minimum-spanning-tree/tutorial/)

### Videos:

For quick and dirty understanding of Kruskal’s and Prim’s algorithms, the following short videos are good.

* [Prim's Algorithm in 2 minutes](https://www.youtube.com/watch?v=cplfcGZmX7I)
* [Kruskal's Algorithm in 2 minutes](https://www.youtube.com/watch?v=71UQH7Pr9kU)

For more details, Graph Theory Playlist of William Fiset is very good. (We don’t cover the Union Find operation for efficiently performing Kruskal’s Algorithm, and we do the Eager version of Prim’s Algorithm).

* [Union Find Kruskal's Algorithm](https://www.youtube.com/watch?v=JZBQLXgSGfs)
* [Prim's Minimum Spanning Tree Algorithm (lazy implementation)](https://www.youtube.com/watch?v=jsmMtJpPnhU)
* [Eager Prim's Minimum Spanning Tree Algorithm](https://www.youtube.com/watch?v=xq3ABa-px_g&t=2s)